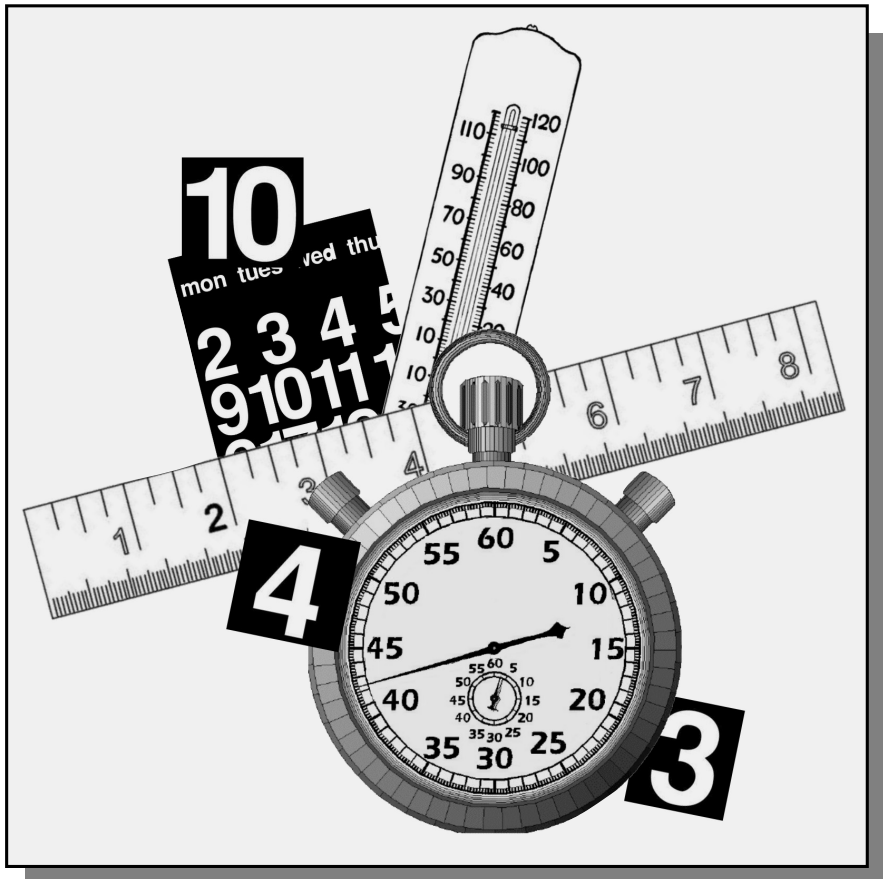
Intermediate Math Skills



Chapter 3

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Intermediate Math Skills

Chapter 3

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"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



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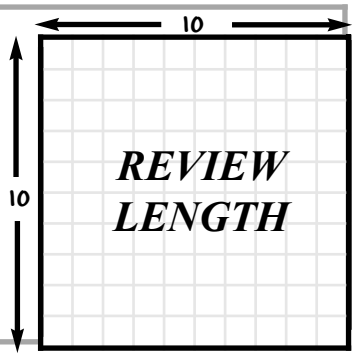
Intermediate Math Skills



Lesson 1

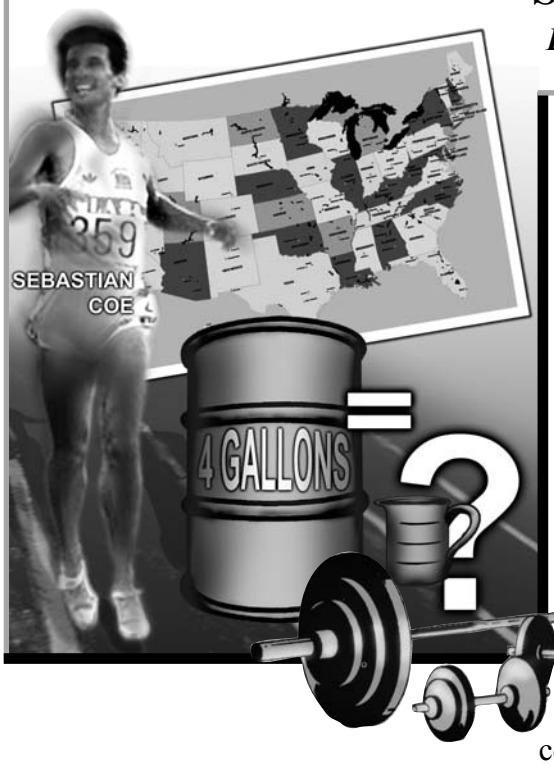
Converting

Chapter 3



Section 1

Lessons 1-5



In this chapter, Professor K will review all that you have learned about applying many of the mathematical formulas and principles to real life situations. Professor K will begin this lesson with a review of ***length*** measurements. Throughout this chapter, you may refer to the Mathematics Chart for formulas and ***conversions*** provided at the end of this chapter.



DEFINITION: LENGTH

Length identifies how long something is.

Remember, if you set up the *conversion* factors correctly, the units will divide out. This process helps you know that you are multiplying and dividing the conversion factors together properly. Look at the examples as Professor K converts various units of length into other units.

Example Set 1

After 9-11-2002, Richard joined the U.S. Marines. During boot camp Richard and his fellow recruits were required to run five miles before breakfast each morning. Just for fun, Richard wanted to calculate his distance in inches for the week. Follow along as Professor K shows you how Richard calculated the number of inches he had run in a week.



Step 1

To calculate the answer, look at the Mathematics Chart and locate the following conversions: 1 mile = 5,280 feet, 1 foot = 12 inches, and 1 week = 7 days.



Length: identifies how long something is

Conversion: a method of changing one unit of measurement into another unit of measurement

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Step 2 Richard then set up the conversions as follows so that the units will divide out.

$$\frac{5\text{ mile}\bullet 5280\text{ ft}\bullet 12\text{ in}\bullet 7\text{ days}\bullet 1}{1\bullet 1\text{ mile}\bullet 1\text{ ft}\bullet 1\text{ week}\bullet 1\text{ day}}$$



Step 3 Richard then performed the calculations following the rules for multiplying fractions.

$$\frac{5\text{ mile}\bullet 5280\text{ ft}\bullet 12\text{ in}\bullet 7\text{ days}\bullet 1}{1\bullet 1\text{ mile}\bullet 1\text{ ft}\bullet 1\text{ week}\bullet 1\text{ day}}=\frac{5\ \cancel{\text{mile}}\bullet 5280\ \cancel{\text{ft}}\bullet 12\text{ in}\bullet 7\ \cancel{\text{day}}\bullet 1}{1\bullet 1\ \cancel{\text{mile}}\bullet 1\ \cancel{\text{ft}}\bullet 1\text{ week}\bullet 1\ \cancel{\text{day}}}$$

Step 4 After simplifying the fraction and dividing the units, Richard then performed the following calculation to determine the number of inches he ran in a week:

$$\frac{5\bullet 5280\bullet 7\bullet 12\text{ inches}}{1\text{ week}}=2,217,600\text{ inches}/\text{week}$$

Example Set 2

In the 1980 and 1984 Olympics, Sebastian Coe from the United Kingdom competed in the 800 m and 1500 m runs. Calculate the difference in length of these two races and express the answer in millimeters.

Step 1 To calculate the answer, you must first find the difference of 1500 m and 800 m. Difference means subtraction, so 1500 m - 800 m = 700 m. Look at the Mathematics Chart and locate the following conversions: 1 meter = 100 centimeters, 1 centimeter = 10 millimeters.

Step 2 You must set up the conversions as follows so that the units will divide out.

$$\frac{700\text{ m}}{1}\bullet\frac{100\text{ cm}}{1\text{ m}}\bullet\frac{10\text{ mm}}{1\text{ cm}}$$

Step 3 You must perform the calculations following the rules for multiplying fractions.

$$\frac{700\text{ m}\bullet 100\text{ cm}\bullet 10\text{ mm}}{1\bullet 1\text{ m}\bullet 1\text{ cm}}=\frac{700\ \cancel{\text{m}}\bullet 100\ \cancel{\text{cm}}\bullet 10\text{ mm}}{1\bullet\ \cancel{\text{m}}\bullet\ \cancel{\text{cm}}}$$

Step 4 After simplifying the fraction and dividing the units, perform the following calculation to determine the difference of the two distances in millimeters.

$$700\bullet 100\bullet 10\text{ mm}=700{,}000\text{ mm}$$

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Show Professor K that you understand how to convert units of length by working the activities for this lesson.

LIFE PRINCIPLE



"All human wisdom is summed up in two words: wait and hope."

—Alexandre Dumas (1802-1870) Writer

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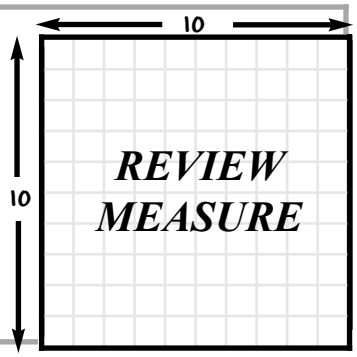


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Lesson 2

Converting

Chapter 3



Section 1

Lessons 1-5



Professor K will review measurements in this lesson. Remember to refer to the Mathematics Chart for formulas and ***conversions*** provided at the end of this chapter. This lesson reviews conversions between the ***English System*** and the ***Metric System*** of measurement.



Professor K

Definition: Volume

Volume is the amount of space something occupies.

Remember, if you set up the conversion factors correctly, the units will divide out. This process helps you to know that you are multiplying and dividing conversion factors together properly. Look at the examples to see how Professor K converts units of volume.

VOCABULARY

Conversion: a method of changing one unit of measurement into another unit of measurement
English System: system of measurement used primarily in the United States; also known as *customary* or *U.S. System*
Metric System: system of measurement used by nearly every county of the world
Volume: the amount of space something occupies

Example Set 1

Professor K needed to mix 4 gallons of organic fertilizer to spray on his garden. Professor K wanted to know how many cups of fertilizer mixture he would have when the mixture was complete. Follow along as Professor K shows you how to calculate the quantity of cups in 4 gallons.

Step 1

To calculate the answer, look at the Mathematics Chart and locate the following conversions: 1 gallon = 4 quarts, 1 quart = 2 pints, and 1 pint = 2 cups.

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Step 2 Professor K then sets up the conversions as follows so that the units will divide out.

$$\frac{4\text{ gal}}{1}\bullet\frac{4\text{ qt}}{1\text{ gal}}\bullet\frac{2\text{ pt}}{1\text{ qt}}\bullet\frac{2\text{ cups}}{1\text{ pt}}$$



Step 3 Professor K then performs the calculations following the rules for multiplying fractions.

$$\frac{4\text{ gal}\bullet 4\text{ qt}\bullet 2\text{ pt}\bullet 2\text{ cups}}{1\bullet 1\text{ gal}\bullet 1\text{ qt}\bullet 1\text{ pt}}=\frac{4\ \cancel{\text{gal}}\bullet 4\ \cancel{\text{qt}}\bullet 2\ \cancel{\text{pt}}\bullet 2\text{ cups}}{1\ \cancel{\text{gal}}\bullet\cancel{\text{qt}}\bullet\cancel{\text{pt}}}$$

Step 4 After simplifying the fraction and dividing the units, Professor K then performs the following calculation.

$4\bullet 4\bullet 2\bullet 2\text{ cups}=16\bullet 2\bullet 2\text{ cups}=32\bullet 2\text{ cups}=64\text{ cups}.$

Professor K has calculated that he would have 64 cups of fertilizer.

Example Set 2

A pharmacist received 3 liters of allergy medicine in a shipment of supplies. Doctors usually prescribed this medicine in doses of 2 ml each. Calculate how many doses could the pharmacist sell from the three-liter supply that he had received. Follow along as Professor K shows you how to calculate how many doses of 2 ml each are in 3 liters.

Step 1 To calculate the answer, look at the Mathematics Chart and locate the following conversion: 1 liter = 1,000 milliliters.

Step 2 You must then set up the conversions as follows so that the units will divide out.

$$\frac{3\text{ liter}}{1}\bullet\frac{1{,}000\text{ ml}}{1\text{ liter}}\bullet\frac{1\text{ dose}}{2\text{ ml}}$$

Step 3 Richard then performed the calculations following the rules for multiplying fractions.

$$\frac{3\text{ liter}\bullet 1{,}000\text{ ml}\bullet 1\text{ dose}}{1\bullet 1\text{ liter}\bullet 2\text{ ml}}=\frac{3\ \cancel{\text{liter}}\bullet 1{,}000\ \cancel{\text{ml}}\bullet 1\text{ dos}}{1\ \cancel{\text{liter}}\bullet 2\ \cancel{\text{ml}}}$$

Step 4 After simplifying the fraction and dividing the units, Richard then performed the following calculation.

$^{3000}/_{2}=3{,}000\div 2=1{,}500\text{ doses}.$

So Professor K has calculated that the pharmacist would be able to prepare 1,500 doses of allergy medicine from a 3-liter bottle.

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Show Professor K that you understand how to convert units of volume by working the activities for this lesson.

Life Principle



"You cannot escape the responsibility of tomorrow by evading it today."

—Abraham Lincoln

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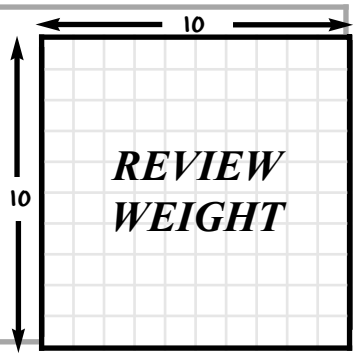


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Lesson 3

Converting

Chapter 3



Section 1

Lessons 1-5



Professor K will review ***weight*** measurements in this lesson. Remember to refer to the Mathematics Chart for formulas and conversions provided at the end of this chapter.

DEFINITION: WEIGHT

Weight is with how much force an object is being pulled toward the center of the earth by gravity.

Remember, if you set up the conversion factors correctly, the units will divide out. This process helps you to know that you are multiplying and dividing the conversion factors together properly. Look at the examples to see how Professor K converts units of weight.

Example Set 1

At the 2000 Olympics in Sidney, Australia, U.S. weightlifting champion Shane Hamman lifted 429.25 pounds. With this lift, Hamman set a new U.S. record. Professor K shows you in this example how to calculate how many ounces Hamman lifted.

Step 1

To calculate the answer, look at the Mathematics Chart and locate the following conversions: 1 pound = 16 ounces.

Step 2

You must set up the conversions as follows so that the units will divide out.

$$\frac{429.25 \text{ pounds}}{1} \bullet \frac{16 \text{ ounces}}{1 \text{ pound}}$$

VOCABULARY

Weight: the amount of force of gravity on an object

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Step 3 You must then perform the calculations following the rules for multiplying fractions.

$$\frac{429.25 \text{ pounds} \bullet 16 \text{ ounces}}{1 \bullet 1 \text{ pound}} = \frac{429.25 \ \cancel{\text{pounds}} \bullet 16 \text{ ounces}}{1 \ \cancel{\text{pound}}}$$

Step 4 After simplifying the fraction and dividing the units, you must perform the following calculation.

$$429.25 \bullet 16 \text{ ounces} = 6{,}868 \text{ ounces.}$$

Professor K has determined that Mr. Hamman lifted 6,868 ounces.

Example Set 2

In **Example Set 1**, you learned that U.S. weightlifting champion Shane Hamman lifted 429.25 pounds at the 2000 Olympics in Sidney, Australia. 429.25 pounds is 195 kilograms. Professor K will show you how to calculate how many milligrams Mr. Hamman lifted.

Step 1 To calculate the answer, look at the Mathematics Chart and locate the following conversions: 1 kilogram = 1,000 grams, 1 gram = 1,000 milligrams.

Step 2 You must set up the conversions as follows so that the units will divide out.

$$\frac{195 \text{ kg}}{1} \bullet \frac{1{,}000 \text{ g}}{1 \text{ kg}} \bullet \frac{1{,}000 \text{ mg}}{1 \text{ g}}$$

Step 3 You must then perform the calculations following the rules for multiplying fractions.

$$\frac{195 \text{ kg} \bullet 1{,}000 \text{ g} \bullet 1{,}000 \text{ mg}}{1 \bullet 1 \text{ kg} \bullet 1 \text{ g}} = \frac{195 \ \cancel{\text{kg}} \bullet 1{,}000 \ \cancel{\text{g}} \bullet 1{,}000 \text{ mg}}{1 \ \cancel{\text{kg}} \bullet \cancel{\text{g}}}$$

Step 4 After simplifying the fraction and dividing the units, you must perform the following calculation.

$$195 \bullet 1{,}000 \bullet 1{,}000 \text{ milligrams} = 195{,}000 \bullet 1{,}000 \text{ milligrams} = 195{,}000{,}000 \text{ milligrams.}$$

Professor K has determined that Mr. Hamman lifted 195,000,000 milligrams.



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Show Professor K that you understand how to convert units of weight by working the activities for this lesson.

Life Principle



"We are what we repeatedly do.
Excellence then, is not an act,
but a habit."
—Aristotle (384-322 BC)

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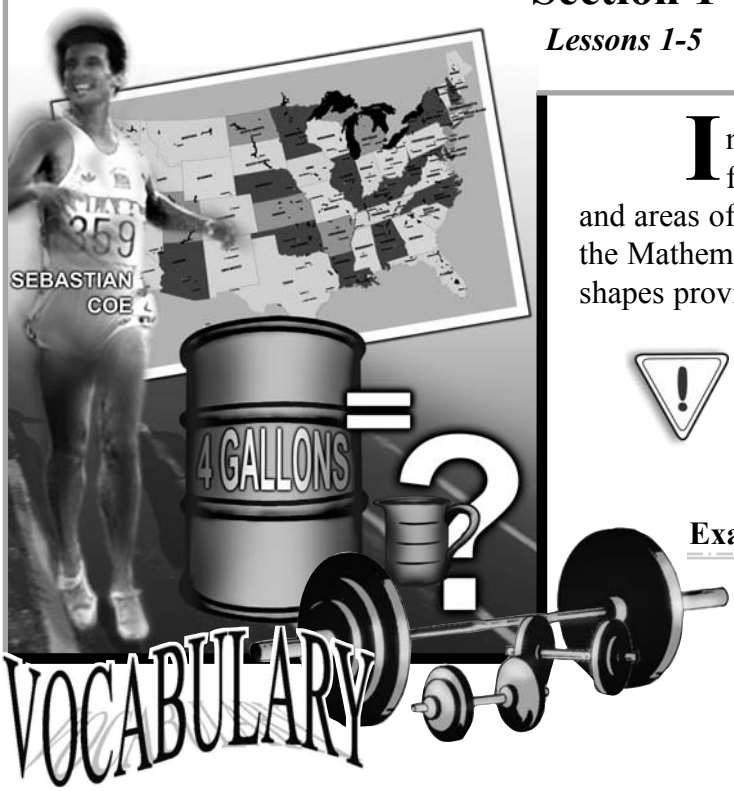
Lesson 4

Geometry

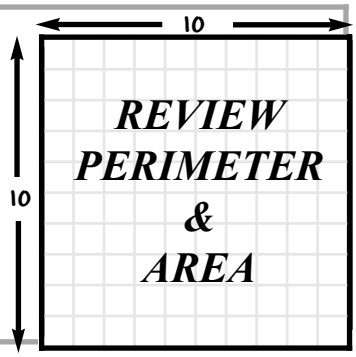
Chapter 3

Section 1

Lessons 1-5



Perimeter: distance around the outside of something
Circumference: the distance around, or perimeter of a circle
Radius: a line segment between the point that is the center of a circle and a point on the circle
Diameter: a line segment whose end points are on the circle, and the center of the circle is also a point on the line
Trapezoid: a 4-sided, plane figure having two parallel and two non-parallel sides

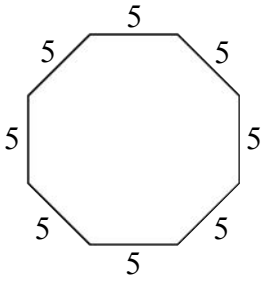


In this lesson, Professor K will review formulas for calculating the ***perimeters*** and areas of various shapes. Remember to refer to the Mathematics Chart for formulas of various shapes provided at the end of this chapter.

DEFINITION: PERIMETER

The perimeter of a polygon is determined by finding the sum of the length of all the sides.

Example Set 1



In the *octagon* (eight-sided figure) above, the length of each side is 5 units. The perimeter is calculated by adding the length of all the sides together. Therefore, the result is the following equation.

perimeter = 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40 units

OR perimeter = 8(sides) × 5(units) = 40 units

The perimeter, or distance around a *circle* is known as the ***circumference***. Professor K refers you to the formula for circumference from the Mathematics Chart at the end of this chapter, and then calculates the circumference of a circle for you in **Example Set 2**.

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Example Set 2

Look at the following explanation to find the circumference of a circle with a ***radius*** of 2 inches, and a ***diameter*** of 4 inches.

First, using the formula for circumference that includes r for radius (and remembering that π equals 3.14) we have:

$$C = 2\pi r = 2(3.14)(2) = 6.28(2) = 12.56 \text{ inches}$$

Next, using the formula for circumference of a circle with a diameter, 4 inches, we have:

$$C = \pi d = \pi(4) = 3.14(4) = 12.56 \text{ inches.}$$

Mrs. K wanted a new rug in her living room. The K's living room measured 30 feet in length and 25 feet wide. Look at **Example Set 3** to see how Professor K calculated how many square feet of carpet Mrs. K needed to purchase.

Example Set 3

Professor K used the formula for the area of a rectangle: $A = l \times w$. He then substituted the values for the dimensions length and width into the formula: $l = 30$ ft, and $w = 25$ ft.

$$A = l \times w = 30 \text{ ft} \times 25 \text{ ft} = 750 \text{ square feet} = 750 \text{ ft}^2$$



Professor K's neighbor wanted to purchase a circular, above-ground swimming pool from a local pool company. The neighbor wanted to know how much area in his backyard the pool would cover if the diameter of the pool was 20 feet. Look at **Example Set 4** to see how Professor K calculated the area for his neighbor.

Example Set 4

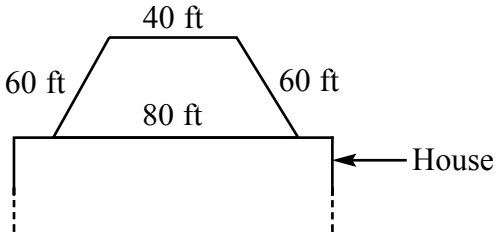
Professor K used the formula for the area of a circle: $A = \pi r^2$. Professor K knew that the radius, *r,* is one-half the diameter; therefore, $r = {}^{d}/_{2} = {}^{20}/_{2} = 10$ feet. He then substituted the value for the radius into the formula: $r = 10$ ft.

$$A = \pi r^2 = \pi \times (10 \text{ ft})^2 = \pi(100 \text{ square feet}) = 3.14(100 \text{ ft}^2) = 314 \text{ ft}^2$$

Professor K's neighbor also wanted to build a privacy fence around his backyard after his pool was delivered. Look at **Example Set 5** to see how Professor K helped his neighbor calculate the length of fence (perimeter of the yard) and the area that the fence would encompass. From the house to the back of the yard is 50 feet.

Example Set 5

Professor K looked at his neighbor's yard and determined that the yard was in the shape of a ***trapezoid***. Professor K drew a sketch of the yard and added the dimensions to the drawing.



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Step 1 Professor K knew that his neighbor wanted only a fence with three sides, so the perimeter equals 160 feet.

$$P = 60 \text{ ft} + 60 \text{ ft} + 40 \text{ ft} = 160 \text{ feet}$$

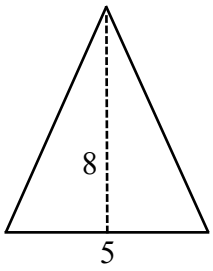
Step 2 Professor K used the formula for area of a trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$. He then substituted the values for base one, $b_1 = 80$ ft and base two, $b_2 = 40$ ft and the height, $h = 50$ ft into the formula.

$$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(80 \text{ ft} + 40 \text{ ft})(50 \text{ ft}) = \frac{1}{2}(120 \text{ ft})(50 \text{ ft}) =$$
$$(\frac{120}{2} \text{ ft})(50 \text{ ft}) = (60 \text{ ft})(50 \text{ ft}) = 3000 \text{ ft}^2$$

Professor K's wife wanted a throw rug in the shape of a triangle. The base of the rug in the shape of a triangle had a length of 5 feet and the height of the triangle had a length of 8 feet. Look at **Example Set 6** to see how Professor K determined how much area the rug would cover.

Example Set 6

Professor K drew a sketch of the rug and added the dimensions.



Next, Professor K used the formula for area of a triangle: $A = \frac{1}{2}bh$.

Professor K then substituted the values for the base, $b = 5$ ft and the height, $h = 8$ ft into the formula.

$$A = \frac{1}{2}bh = \frac{1}{2}(5 \text{ ft})(8 \text{ ft}) = \frac{1}{2}(40\text{ft}^2) = \frac{40}{2} \text{ ft}^2 = 20 \text{ ft}^2$$

Show Professor K that you understand how to calculate the perimeter and area of shapes by working the activities for this lesson.

Life Principle



"If you kick a stone in anger,
you'll hurt your own foot."
—Korean Proverb

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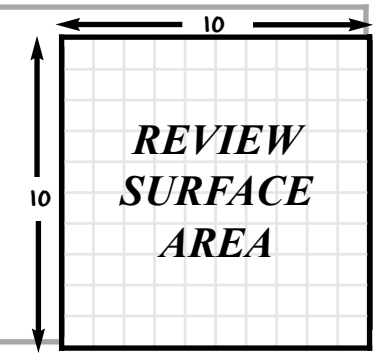


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Geometry

Lesson 5

Chapter 3



Section 1

Lessons 1-5



In the previous lesson, Professor K reviewed with you the calculation of perimeters and areas of various shapes. In this lesson, Professor K will review formulas for calculating the ***surface area*** of various solid shapes. Remember to refer to the Mathematics Chart for formulas of various surface areas provided at the end of this chapter.



Professor K

Professor K wanted to build a toy box in the shape of a cube for his young son. The edge of the toy box was to be 2 feet in length. Look at **Example Set 1** to see how Professor K calculated the amount of wood he would need to construct the cubical toy box.

Example Set 1

Professor K remembers that a cube has six sides, therefore, he could add the area of each side together or he can go to the Mathematics Chart and locate the formula for calculating the surface area of a cube, $A_s = 6s^2$. Professor K substitutes the value of the length of an edge or side (s) into the formula and performs the calculation.

$A_s = 6(2 \text{ feet})^2 = 6 \bullet (2 \bullet 2) \text{ ft}^2 = 6 \bullet 4 \text{ ft}^2 = 24$ square feet

Mrs. K was making a craft project for her Sunday school class. The shape she was making was in the shape of a sphere with a radius of 6 inches. Mrs. K wanted to paint the sphere with craft paint. Look at **Example Set 2** to see how Professor K helped his wife to determine how much paint to purchase.

VOCABULARY

Surface area: the total area of all the areas of the faces of a polyhedron; the area of the surface of a sphere

Lateral: having to do with the side(s) of an object

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Example Set 2

Professor K read from a one fluid ounce bottle of craft paint that one fluid once of paint would cover 100 square inches of surface. Professor K looked at the Mathematics Chart and located the formula for calculating the surface area of a sphere, $A_s = 4\pi r^2$. Professor K substituted the value of the radius, *r,* into the formula and performed the calculations.

$A_s = 4\pi r^2 = 4 \bullet (3.14) \bullet (6 \text{ in.})^2 = 12.56 \bullet (6 \bullet 6) \text{ sq in.} = 12.56 \bullet 36 \text{ sq in.} = 452.16 \text{ sq in.}$

Professor K determined that the surface area of the sphere was 452.16 square inches. To determine the number of bottles of craft paint that Mrs. K would have to purchase, Professor K then divided the total surface area, 452.16 in^2 by the amount of area each bottle would cover: 100 in^2.

$$456.16 \text{ in}^2 \div \frac{100 \text{ in}^2}{1 \text{ bottle}} = \frac{452.16 \text{ in}^2}{1} \bullet \frac{1 \text{ bottle}}{100 \text{ in}^2} = \frac{452.16 \text{ in}^2 \bullet 1 \text{ bottle}}{1 \bullet 100 \text{ in}^2} = 4.5216 \text{ bottles}$$

Professor K determined that his wife would need over four and one-half bottles of craft paint to complete her project. Therefore, she would have to purchase five bottles of craft paint.

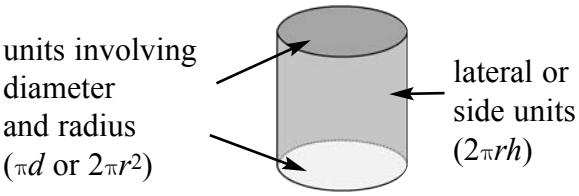
Mrs. K also wanted to paint a popcorn tin to place on her kitchen counter. The popcorn tin was 12 inches tall, eleven inches across (diameter) and approximately 31 inches around. Look at **Example Set 3** to see how Professor K calculated the total surface area of the popcorn tin. (Remember, the radius is one half of the diameter.)

Example Set 3

Professor K knew that the popcorn tin was cylinder shaped. Professor K looked at the Mathematics Chart and located the formula for calculating the total surface area of a cylinder, $A_s = 2\pi rh + 2\pi r^2$. Professor K substituted the value of the radius, $r = 11$ (diameter) $\div 2 = 5.5$ in and the height, $h = 12$ inches into the formula and performed the calculations.

$A_s = 2\pi rh + 2\pi r^2 = 2 \bullet (3.14) \bullet (5.5 \text{ in.}) \bullet (12 \text{ in.}) + 2 \bullet (3.14) \bullet (5.5 \text{ inches})^2 =$
$414.48 \text{ in}^2 + 34.54 \text{ in}^2 = 449.02 \text{ in}^2$

Professor K determined that the total surface area of the popcorn tin was 449.02 square inches. Finding only the ***lateral*** units means using only the part of the formula pertaining to the sides.



Mrs. K was giving her son a birthday party. She was making paper party hats in the shape of cones. Mrs. K asked her husband to calculate how much paper she would need for 20 hats, if the diameter of each hat was 10 centimeters across and 15 centimeters long. Look at **Example Set 4** to see how Professor K made this calculation.



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Example Set 4

Professor K located the formula for calculating the lateral surface area of a cone, $A_{ls} = \pi rl$. (Professor K used the formula for the lateral surface area, not the formula for the total surface area, because Mrs. K was not covering the circular base of the cone.) Professor K substituted the value of the radius, r = diameter $\div$ 2 = 10 $\div$ 2 = 5 centimeters and the lateral length (length along the side), l = 15 centimeters into the formula and performed the calculations.



$A_{ls} = \pi rl = (3.14) \bullet (5 \text{ cm}) \bullet (15 \text{ cm}) = 15.7 \text{ cm} \bullet 15 \text{ cm} = 235.5 \text{ cm}^2$

Professor K determined that the surface area of the party hats was 235.5 square centimeters.

Show Professor K that you understand how to calculate the surface area of solid shapes by working the activities for this lesson.

LIFE PRINCIPLE



"Don't let yesterday use up too much of today."

—Native American Proverb

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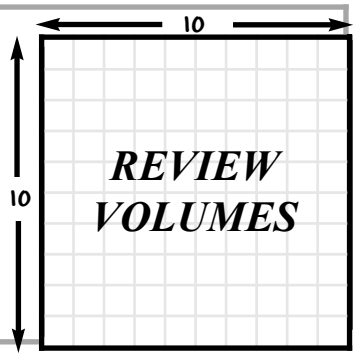
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Lesson 6

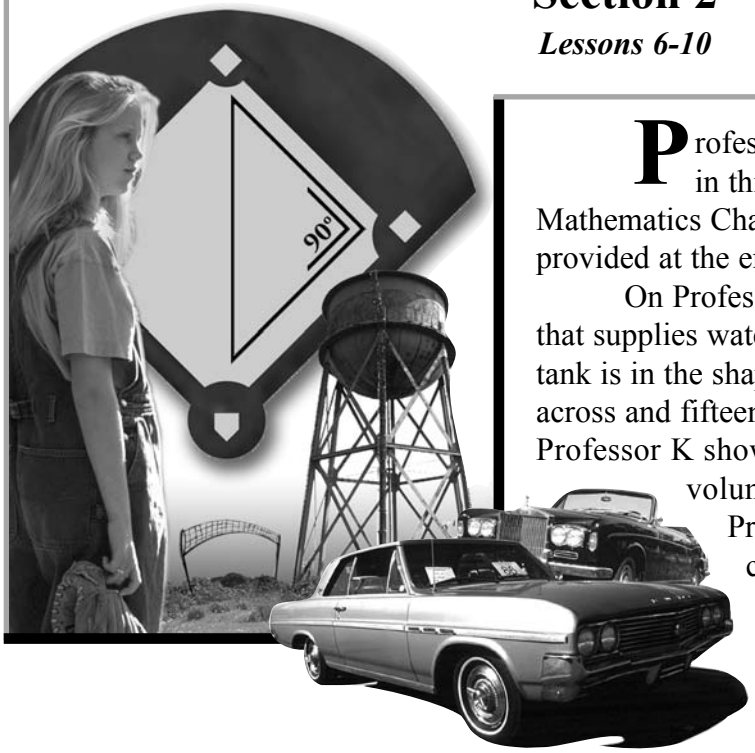
Geometry

Chapter 3



Section 2

Lessons 6-10

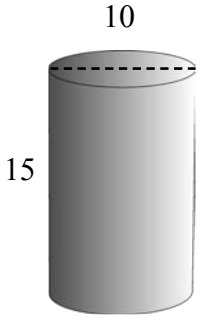


Professor K will review ***volume*** of shapes in this lesson. Remember to refer to the Mathematics Chart for formulas and conversions provided at the end of this chapter.

On Professor K's farm is a water storage tank that supplies water to Professor K's house. The storage tank is in the shape of a cylinder. The tank is ten feet across and fifteen feet tall. Look at **Example Set 1** as Professor K shows you how he calculated the volume of the storage tank in cubic feet. Professor K then shows you how to convert cubic *feet* to *gallons*.

Example Set 1

Professor K made a drawing of the water tank.





Volume: the amount of space something occupies
Capacity: amount that an object will hold; calculated the same as volume



The formula for the volume of a cylinder is: $V = Bh$. The B in the formula is the area of the circular base or end: $B = A = \pi r^2$. Therefore, the formula for the volume of a cylinder becomes, $V = \pi r^2 h$.

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Step 2 Professor K substituted the dimensions of the water storage tank ($r = {}^{10}/_{2} = 5$ feet; and $h = 15$ feet) and the value of π into the formula.

$V = \pi r^2 h = 3.14 \bullet (5 \text{ feet})^2 \bullet 15 \text{ feet} = 3.14 \bullet 25 \text{ ft}^2 \bullet 15 \text{ feet} =$
$78.5 \text{ ft}^2 \bullet 15 \text{ ft} = 1177.5 \text{ cubic feet}$

Step 3 Professor K knew that one cubic foot of water contains approximately 7.5 gallons. Professor K converted the answer in cubic feet to gallons.

$1177.5 \text{ ft}^3 \bullet 7.5 \text{ gal} = 8831.25 \text{ gallons}$

The town near Professor K's farm has a large water storage tank in the shape of a sphere sitting on top of a platform high above the ground. Your town most likely has one like this, too. This storage tank has a diameter of 60 feet. Look at **Example Set 2** as Professor K shows you how to calculate the ***capacity*** of the storage tank in gallons.

Example Set 2

Professor K made a drawing of the water tank.





Step 1 Professor K located the formula for the volume of a sphere in the Mathematics Chart: $V = {}^{4}/_{3}\pi r^3$.

Step 2 Professor K substituted the dimensions of the water storage tank ($r = {}^{60}/_{2} = 30$ feet) and the value of π into the formula.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \bullet 3.14 \bullet (30 \text{ ft})^3 = \frac{12.56}{3} \bullet 27{,}000 \text{ ft}^3 = 12.56 \bullet 9{,}000 \text{ ft}^3 = 113{,}040 \text{ ft}^3$$

Step 3 Professor K knew that one cubic foot of water contained approximately 7.5 gallons. Using that data, Professor K converted the answer in cubic feet to gallons.

$$\frac{113{,}040 \text{ ft}^3}{1} \bullet \frac{7.5 \text{ gal}}{1 \text{ ft}^3} = \frac{113{,}040 \text{ } \cancel{\text{ft}^3} \bullet 7.5 \text{ gal}}{1 \bullet 1 \text{ } \cancel{\text{ft}^3}} = 847{,}800 \text{ gal}$$

Calculating the volume of other solid objects is a simple procedure to by substituting the values of the dimensions into the formulas and performing the calculations.

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Show Professor K that you understand how to calculate the volume of various solids by working the activities for this lesson.

Life Principle



"Things turn out best for the people who make the best out of the way things turn out."

—Art Linkletter,
Television and radio personality

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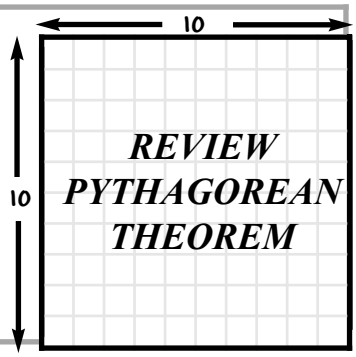
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Lesson 7

Algebra

Chapter 3



Section 2

Lessons 6-10



Professor K will review the Pythagorean Theorem in this lesson. Remember to refer to the Mathematics Chart (at the end of this chapter) for formulas and conversions.

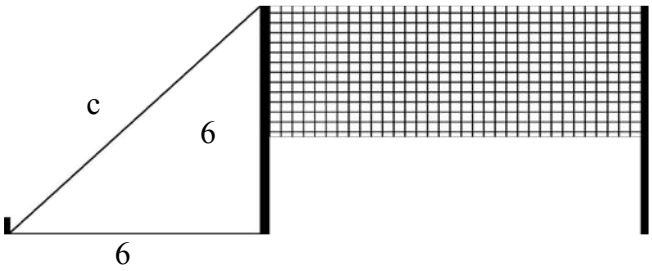


Professor K

Professor K's wife wanted a volleyball net placed in their backyard for the children to play volleyball. The net was twenty feet long and two and one-half feet wide. Professor K installed a post at each end of the net. Then he decided to place support ropes tied to the tops of the posts and to stakes six feet from the bottoms of the posts. The tops of the posts were six feet from the ground. Look at **Example Set 1** as Professor K shows you how he calculated the lengths of the support ropes.

Example Set 1

Professor K began this problem by first drawing a diagram of the net, post, stake and rope.



Step 1

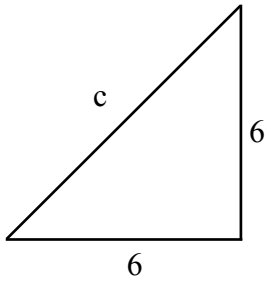
Professor K looked at the triangle formed by the rope, the post and the ground from one post to the stake and constructed the triangle below in relation to the Pythagorean Theorem: "The square of the ***hypotenuse*** of a right triangle equals the sum of the squares of the other two sides."

VOCABULARY

Hypotenuse: the side opposite the 90° angle in a right triangle; the longest side of a right triangle

Radical form: form of a mathematical expression containing a radical sign ($\sqrt{\ }$)

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Step 2

Professor K substituted the dimensions of the triangle into the Pythagorean Theorem (from the Mathematics Chart), where c (the hypotenuse) = length of the support rope, b = 6 ft (height of the post) and a = 6 ft (the distance from the bottom of the post to the stake).

$c^2 = a^2 + b^2 \rightarrow c^2 = 6^2 + 6^2 \rightarrow c^2 = 6 \bullet 6 + 6 \bullet 6 \rightarrow c^2 = 36 + 36 \rightarrow c^2 = 72$

Step 3

Professor K determined that $c^2 = 72$. He then calculated the square root of both sides to determine the value of c, using the following ***radical form***:

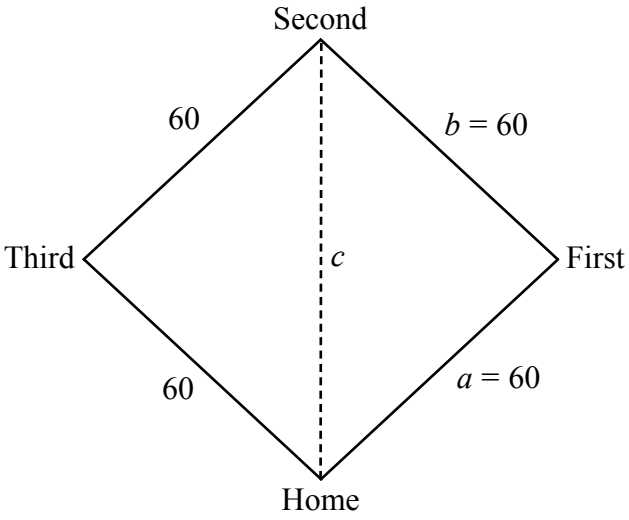
$\sqrt{c^2} = \sqrt{72} \rightarrow c = 8.48$ feet.

Professor K rounded the length of the rope to the nearest tenth to determine that the rope needed to be 8.5 feet to reach from the top of the post to the stake on the ground.

Besides volleyball, Professor K's children enjoy playing baseball in the summer. The baseball diamond that the children use is in the shape of a square with sides of length 60 feet. At each corner of the square is a base. Look at **Example Set 2** to see how Professor K determined the straight-line distance between home plate and second base.

Example Set 2

Professor K began this problem by first drawing a diagram of the baseball diamond.

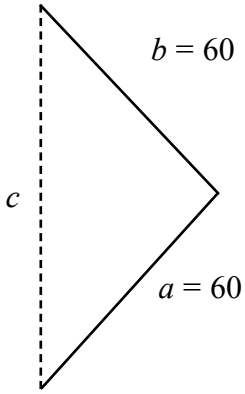




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Step 1

Professor K looked at the triangle formed by the line between home plate and second base, the line between home plate and first base and the line between first and second base. Professor K then constructed the triangle below in relation to the Pythagorean Theorem.



Step 2

Professor K substituted the dimensions of the triangle into the Pythagorean Theorem (from the Mathematics Chart), where c (the hypotenuse) = the distance between home plate and second base, b = 60 ft (distance between first and second bases) and a = 60 ft (the distance between home plate and first base).

$$c^2 = a^2 + b^2 \rightarrow c^2 = 60^2 + 60^2 \rightarrow c^2 = 60 \bullet 60 + 60 \bullet 60 \rightarrow$$
$$c^2 = 3{,}600 + 3{,}600 \rightarrow c^2 = 7{,}200$$

Step 3

Professor K determined that $c^2 = 7{,}200$. He then calculated the square root of both sides to determine the value of c.

$$\sqrt{c^2} = \sqrt{7{,}200} \rightarrow c = 84.9 \text{ feet.}$$

Professor K rounded the distance between home plate and second base to the nearest whole number, to determine that the distance between home plate and second base to be 85 feet.

Show Professor K that you understand how to use the Pythagorean Theorem by working the activities for this lesson.

Life Principle



"Alter your attitude and you can alter your life."

—Anonymous

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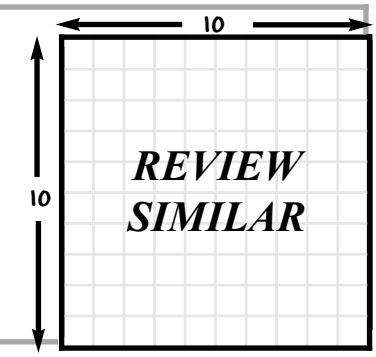


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Geometry

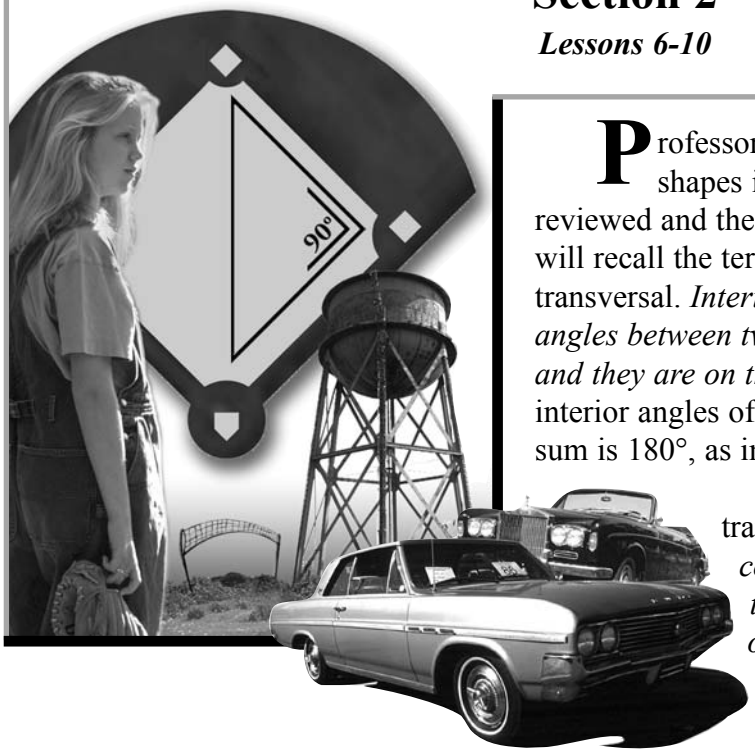
Lesson 8

Chapter 3



Section 2

Lessons 6-10



Professor K will review angles and similar shapes in this lesson. ***Transversals*** will be reviewed and the angles around a transversal. Also you will recall the terminology for the angles around a transversal. *Interior angles* of a transversal are *those angles between two parallel lines cut by a transversal and they are on the same side of the transversal*. When interior angles of a transversal are added together, the sum is 180°, as in angles 3, 5 and 4, 6 in **Figure 8.1**.

The *corresponding angles* of a transversal are *those angles that are in a corresponding position in relationship to two parallel lines and a transversal of those two lines*. All corresponding angles of a transversal are also congruent, ***vertical angles*** as in angles 1, 5 and 3, 7 and 2, 6 and 4, 8.

Alternate interior angles of a transversal are *those angles that are between two parallel lines cut by a transversal and on opposite sides of the transversal*, as in angles 3, 6 and 4, 5 of **Figure 8.1**. Alternate interior angles are congruent.

Alternate exterior angles of a transversal are *angles that are above and below two parallel lines cut by a transversal and on opposite sides of the transversal*, as in angles 1, 8 and 2, 7 in **Figure 8.1**. Alternate exterior angles are congruent.

VOCABULARY

Transversal: a line that intersects two or more parallel lines

Vertical angles: angles on opposite sides of intersecting lines or transversals

Supplementary angles: angles whose sum of their measures equals 180°; form a straight line when joined together

Complementary angles: angles whose sum of their measures equals 90°

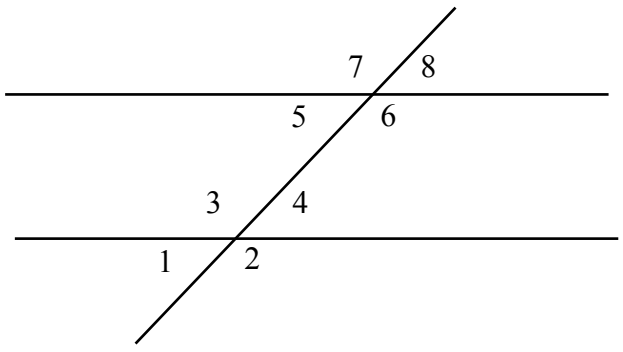


Figure 8.1

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Look at **Example Set 1** as Professor K names the angles formed by a transversal crossing the two parallel lines in **Figure 8.1**.

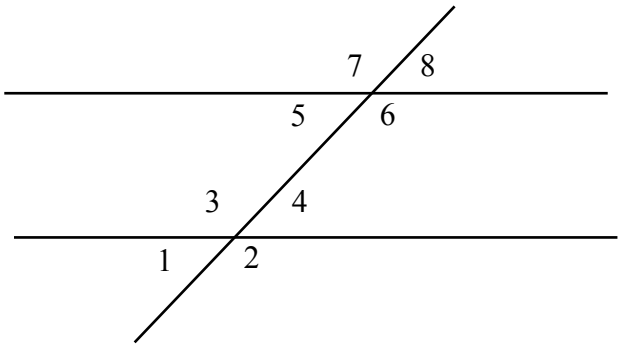




Figure 8.1

Example Set 1

Vertical angles:

$1 \cong 4, 5 \cong 8, 2 \cong 3, 6 \cong 7$

These angles are *vertical angles* because they are angles on opposite sides of intersecting lines. Notice that the measures of vertical angles are equal, and therefore, the angles are congruent.

Supplementary angles:

1 & 3, 2 & 4, 1 & 2, 3 & 4,
5 & 7, 5 & 6, 7 & 8, 6 & 8

These angles are *supplementary angles* because the angles form a straight line when joined together. The sum of the measures of these angles would also equal 180°.

Corresponding angles of a transversal:

1 & 3, 2 & 4, 1 & 2, 3 & 4,
5 & 7, 5 & 6, 7 & 8, 6 & 8

These angles are *corresponding angles* because the angles are in a corresponding position in relationship to two parallel lines and a transversal of those two lines. Corresponding angles are also congruent.

Alternate exterior angles:

1 & 8, 2 & 7

These angles are above and below two parallel lines cut by a transversal and on opposite sides of the transversal. *Alternate exterior angles* are congruent.

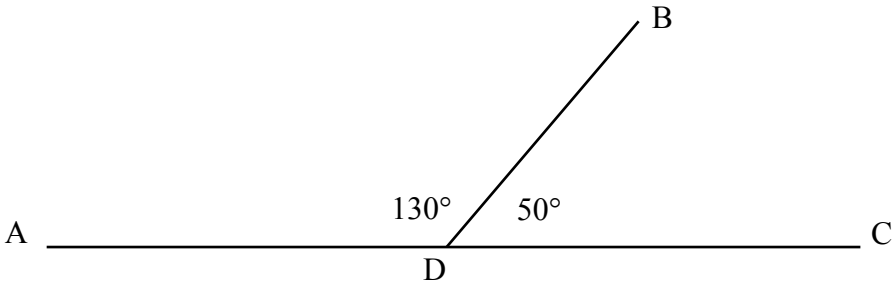
Alternate interior angles:

3 & 6, 4 & 5

These angles are between two parallel lines cut by a transversal and on opposite sides of the transversal. *Alternate interior angles* are congruent.

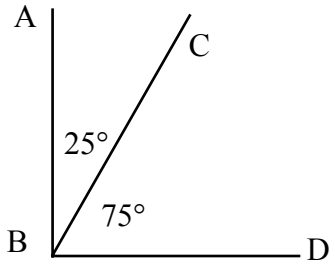
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Example Set 2



m∠ADB = 130°
m∠BDC = 50°
m∠ADB + m∠BDC = 130° + 50° = 180°
∠ADB and ∠BDC are supplementary angles because the sum of their measures is 180°.

Example Set 3



m∠ABC = 25°
m∠CBD = 75°
m∠ABC + m∠CBD = 90°
∠ABC and ∠CBD are ***complementary angles*** because the sum of their measures equals 90°.

Show Professor K that you understand how to recognize various kinds of angles by working the activities for this lesson.

Life Principle



"Days are scrolls; write on them what you want to be remembered."

—Anonymous

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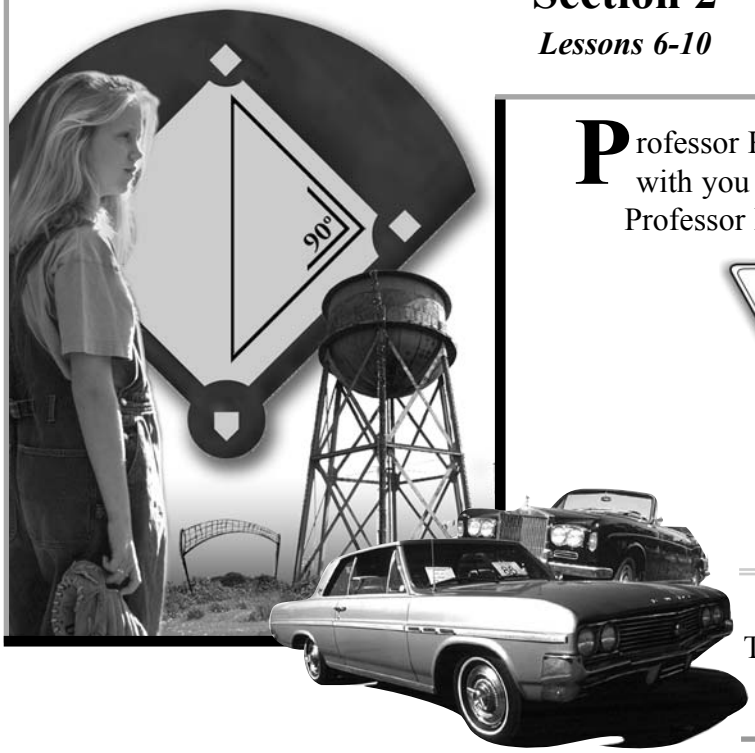
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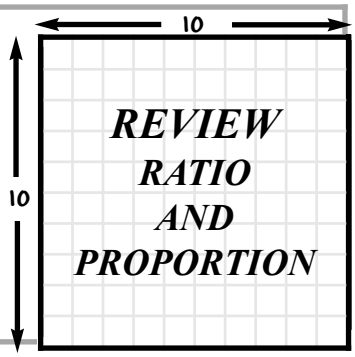
Lesson 9

Chapter 3

Section 2

Lessons 6-10





Professor K will review ***ratios*** and ***proportions*** with you in this lesson.

Professor K reminds you of the following rules.

RULE 1

A proportion is an equation of two ratios separated by an equals sign. Two ratios are proportional if the fractions that represent those ratios are equal.

$^5/_{10} = {}^{50}/_{100}$; $^3/_6 = {}^6/_{12}$; $^2/_{10} = {}^{10}/_{50}$

The ratios above that are separated by equals signs are proportions.

RULE 2

An equation represents a proportion if the products of the cross-multiplications are equal.

$^1/_2 = {}^{50}/_{100} \rightarrow 1 \times 100 = 2 \times 50 \rightarrow 100 = 100$;
ratios are proportional

$^2/_6 = {}^6/_{12} \rightarrow 2 \times 12 = 6 \times 6 \rightarrow 24 \neq 36$;
ratios are not proportional

VOCABULARY

Ratio: a comparison of two numbers

Proportion: a comparison of ratios

Example Set 1

Consider two schools, A and B. School A has a student population consisting of 300 boys and 500 girls. School B has a student population consisting of 200 boys and 400 girls. Professor K will show you how to calculate ratios using the information from these schools.

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a) Professor K calculates the ratio of boys to girls in each school.

$$\text{Ratio of boys to girls in school A: } \frac{300 \text{ boys}}{500 \text{ girls}} = \frac{3 \text{ boys}}{5 \text{ girls}}$$

$$\text{Ratio of boys to girls in school B: } \frac{200 \text{ boys}}{400 \text{ girls}} = \frac{2 \text{ boys}}{4 \text{ girls}} = \frac{1 \text{ boy}}{2 \text{ girls}}$$

b) School C has a student population containing 600 girls. The ratio of boys to girls in School C is the same as the ratio of boys to girls in School B. Professor K calculates the number of boys in School C by setting up a proportion between the students in School B and the students in School C. (Remember a *proportion* is an *equality* between two *ratios*.)

$$\text{(ratio School B) } {}^{1}\!/_{2} = \text{(ratio School C) } {}^{x}\!/_{600} \rightarrow {}^{1}\!/_{2} = {}^{x}\!/_{600}$$

Professor K cross-multiplies to obtain the following equation from the proportion above:

$$2x = 1 \bullet 600 \rightarrow 2x = 600 \rightarrow x = {}^{600}\!/_{2} \rightarrow x = 300.$$

Therefore, Professor K has determined by using proportions that 300 boys are in School C.

In **Example Set 2**, Professor K compares the fuel efficiency of two automobiles by using proportions.

Example Set 2

Professor K considered two cars that he thought he might purchase, cars A and B. Car A had a fuel efficiency rating of 220 miles on a tank of gas. The gas tank held 11 gallons of gas. Car B could travel 300 miles before its 15 gallon tank is empty. Professor K used proportions to calculate the miles per gallon that he could expect from each car.



a) Professor K sets up a proportion for the fuel efficiency of Car A.

$$\frac{220 \text{ miles}}{11 \text{ gal}} = \frac{x \text{ miles}}{1 \text{ gal}}$$

Professor K cross-multiplies to obtain the following equation from the proportion above:

$$220 \bullet 1 = 11x \rightarrow 220 = 11x \rightarrow {}^{220}\!/_{11} = x \rightarrow x = 20.$$

Professor K calculated that Car A's fuel efficiency was 20 miles per gallon.

b) Professor K sets up a proportion for the fuel efficiency of Car B.

$$\frac{300 \text{ miles}}{15 \text{ gal}} = \frac{x \text{ miles}}{1 \text{ gal}}$$

Professor K cross-multiplies to obtain the following equation from the proportion above:

$$300 \bullet 1 = 15x \rightarrow 300 = 15x \rightarrow {}^{300}\!/_{15} = x \rightarrow x = 20.$$

Professor K calculated that Car B's fuel efficiency was 20 miles per gallon.

Therefore, Professor K has determined by using proportions that both cars had the same fuel efficiency, therefore, in regard to fuel efficiency, it did not matter which car he purchased.

Show Professor K that you understand the use of ratios and proportions by working the activities for this lesson.

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Life Principle



"Usually your success is proportional to your effort."

—Anonymous

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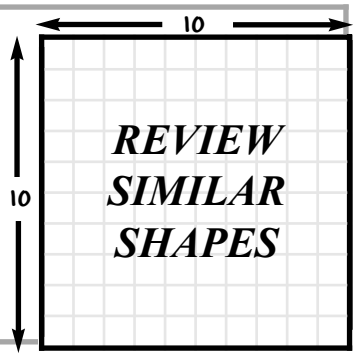
Lesson 10

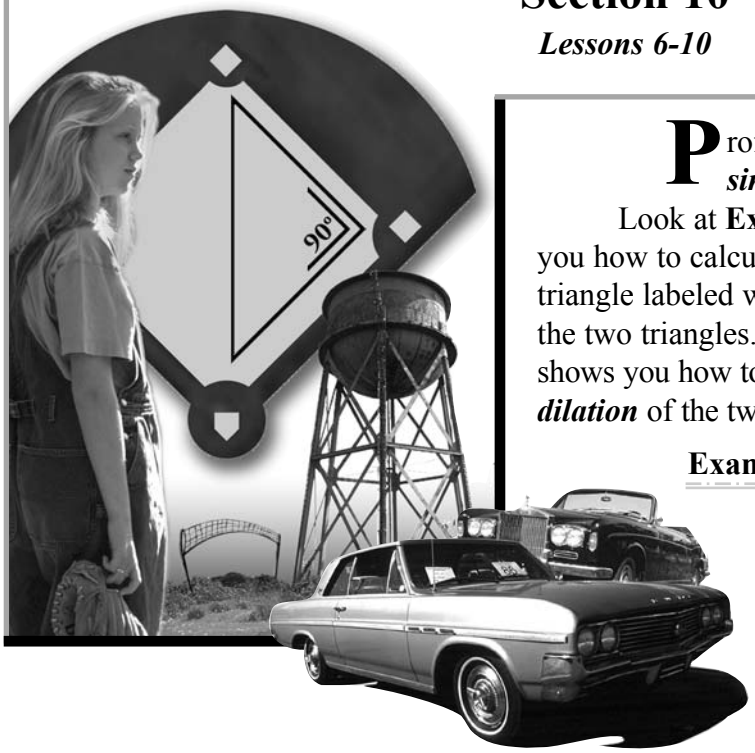
Geometry

Chapter 3

Section 10

Lessons 6-10

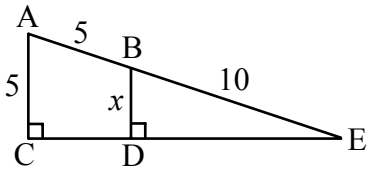




Professor K will review angles and ***similar*** shapes in this lesson.

Look at **Example Set 1** as Professor K shows you how to calculate the value of the leg of the triangle labeled with an *x*, using the similarity between the two triangles. In **Example Set 2**, Professor K shows you how to calculate the factor of reduction or ***dilation*** of the two triangles in **Example Set 1**.

Example Set 1



Notice that these two triangles are similar because both contain corresponding right angles and both contain the ***congruent*** ∠E. By using the same proof as in **Example Set 1** of Chapter 7, Lesson 6, Professor K knows that ∠CAE = ∠DBE. Therefore, △ACE ~ △BDE, because of the A-A-A property of similar triangles. Professor K then sets up the following ratios to determine the length of the line segment BD.

$$\frac{AE}{BE} = \frac{AC}{BD} \rightarrow \frac{15}{10} = \frac{5}{x}$$

Professor K wants you to notice that the length of the line segment AE was determined by adding the length of AB to BE (5 + 10 = 15). Professor K cross multiplies to obtain the equation:

$$\frac{15}{10} = \frac{5}{x} \rightarrow 15x = 5(10)$$

Solving for *x* by dividing both sides of the equation by 15, Professor K determines that $x = 3\frac{1}{3}$.

VOCABULARY

Similar: having the same shape but not necessarily the same size; signified by ~

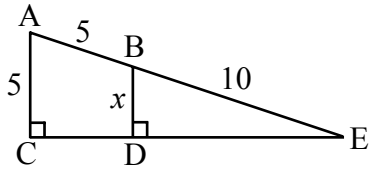
Dilation: the amount of reduction or factor by which a number is reduced or made less

Corresponding: in the same respective location

Congruent: having the same measure

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Example Set 2



Professor K determines the scale factor from ΔACE and ΔBDE in **Example Set 1**. The factor or amount of reduction is calculated by setting up a ratio between the smaller triangle and the larger triangle.

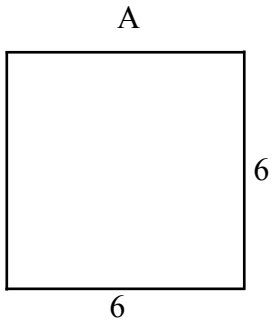
$$BE/AE = 10/15 = 2/3$$

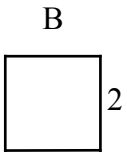
Professor K has calculated the scale factor to be $2/3$. ΔBDE is a $2/3$ *dilation* of ΔACE, or ΔACE is reduced by $2/3$ to obtain ΔBDE.

In **Example Set 3**, Professor K shows you how to determine the scale factor of increase between the areas of two squares.

Example Set 3

The length of the sides of square A is three times the length of the sides of square B. Professor K shows you how to determine the relationship of the area of the squares. Remember, area is determined by multiplying two lengths. Therefore, any relationship between the lengths of the sides to the area would be squared.







The area of square A would be nine times that of square B (the relationship of the length of the sides is a factor of three, therefore the relationship between the areas would be the factor squared, or $3^2 = 9$). Professor K checks this answer by comparing the calculations of the area of each square.

- area of square A = $6 \bullet 6 = 36$ square units
- area of square B = $2 \bullet 2 = 4$ square units

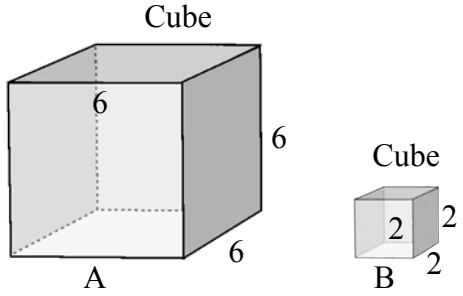
Ratio of the area of square A to square B: $36/4 = 9/1$

In **Example Set 4**, Professor K shows you how to compare the scale factor of the volume of two solids.

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Example Set 4

The length of the edges of cube A is three times the length of the edges of cube B. Professor K shows you how to determine the relationship of the volume of the cubes. Remember, volume is determined by multiplying three lengths. Therefore, any relationship between the lengths of the edges to the volume would be cubed.



The volume of cube A would be twenty-seven times that of cube B (the relationship of the length of the edges is a factor of three, therefore the relationship between the volumes would be the factor cubed, or $3^3 = 27$). Professor K checks this answer by comparing the calculations of the volume of each cube.

- volume of cube A = $6 \bullet 6 \bullet 6 = 216$ cubic units
- area of square B = $2 \bullet 2 \bullet 2 = 8$ cubic units

Ratio of the volume of cube A to cube B: $^{216}/_{8} = {}^{27}/_{1}$

Show Professor K that you understand how to work problems of similar shapes by working the activities for this lesson.

LIFE PRINCIPLE



"Regarding the shape of your life, your choices are part of the equation."

—Anonymous

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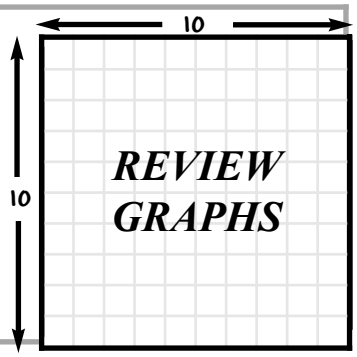
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Lesson 11

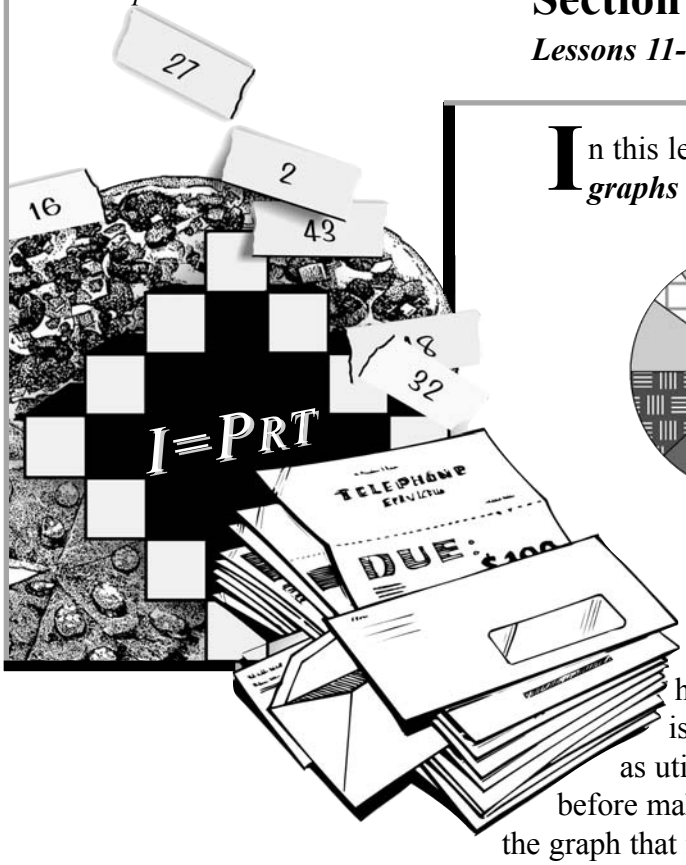
Graphs

Chapter 3



Section 3

Lessons 11-15



In this lesson, Professor K will review various ***graphs*** with you.

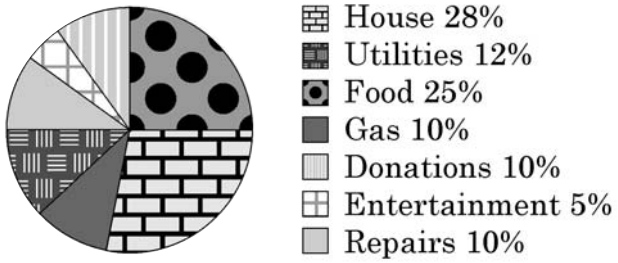


Figure 11.1

Example Set 1

Figure 11.1 is a pie graph of a family's household expenses. Determine which item is just over twice the amount of expenditure as utilities. Professor K looks at the entire graph before making a decision. He locates the portion of the graph that represents "utilities" (12 %). Professor K multiplies 12% by 2 to obtain 24%. The slice of the pie graph that is just over 24% is the "food" slice, 25%. Therefore, the expense item that is just over twice the amount of utilities is food, or said another way, the family spends *twice* as much for *food* as for utilities.

VOCABULARY

Graph: a pictorial representation of the comparison of two or more numbers
x-axis: the horizontal base line in a Cartesian coordinate system
y-axis: the vertical base line in a Cartesian coordinate system

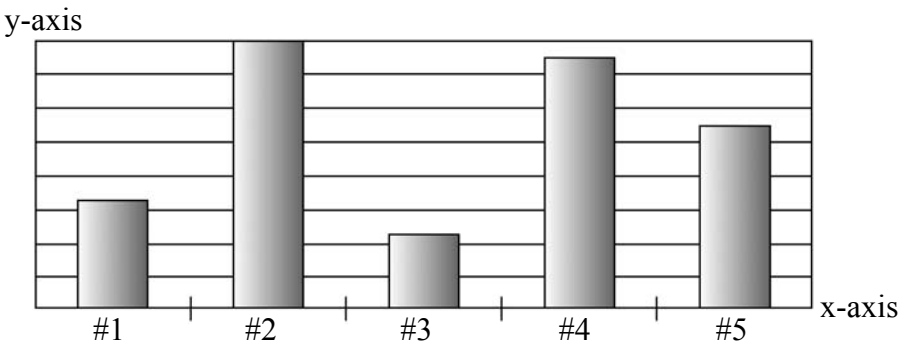


Figure 11.2

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Example Set 2

Figure 11.2 is a column graph of the number of cars five salesmen sold for a local dealership during one week. The ***x-axis*** is the number of each salesman and the ***y-axis*** represents the number of cars sold by each salesman. Determine which salesman sold the most cars during this week and which salesman sold the least. Professor K looks at the graph and determines that salesman #2 sold the most cars because the graph of his car sales is the tallest. Professor K determines that salesman #3 sold the least because the bar representing his sales is the shortest.



Figure 11.3 is the graph of a company's gross income from the company's three regional offices located in the United States. Look at **Example Set 3** to see how Professor K reads this graph. The x-axis represents the quarters of the year and the y-axis represents the amount of income in millions of dollars (*80* means *80 million dollars*).

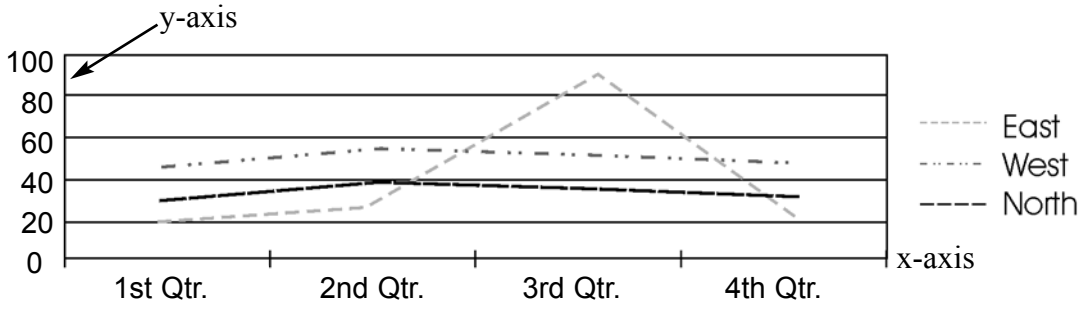


Figure 11.3

Example Set 3

Which quarter(s) did the Eastern regional office have an income of 20 million dollars and which quarter showed a peak of 90 million? Professor K looks at the graph and the legend to the right. He determines which line represents the Eastern office. Professor K then looks at the graph, locates the "East" line, and follows it from the beginning to the end. Professor K determines that from the graph it appears that the Eastern office had sales of 20 million dollars in both the first and fourth quarters, and peaks at 90 million in the third quarter. Professor K points out that you should look at the whole graph and think about the question asked, then make sure you have presented all the possible solutions to the question.

Show Professor K that you understand how to read and interpret graphs by working the activities for this lesson.

Life Principle



"Draw a line between good and bad and do not cross it."

—Anonymous

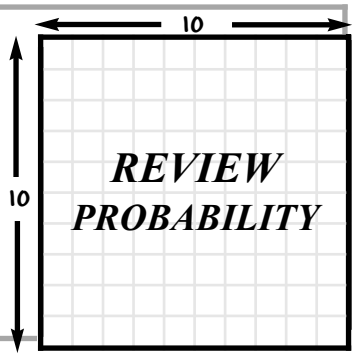
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Lesson 12

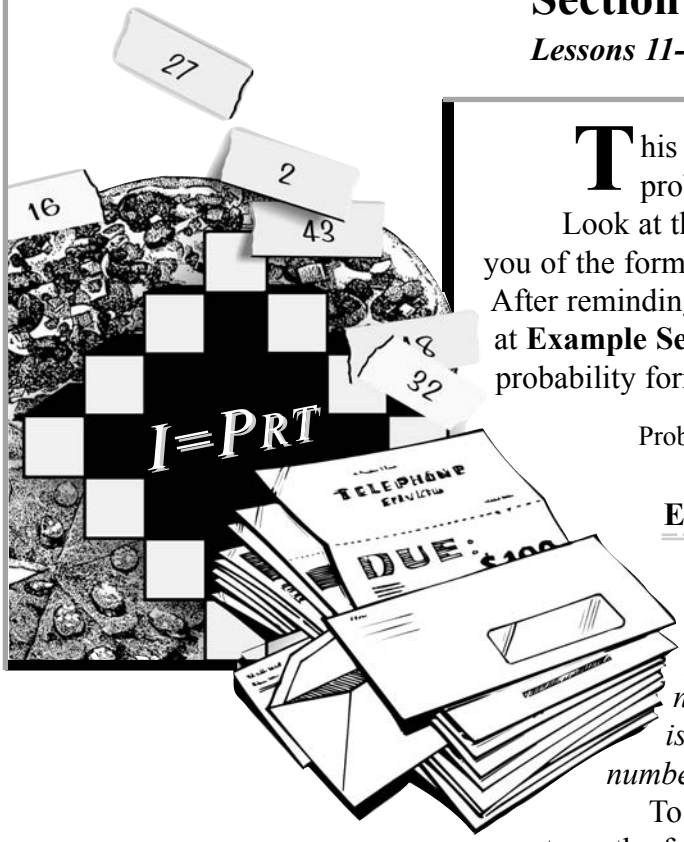
Predictions

Chapter 3



Section 3

Lessons 11-15



This lesson will be a review of principles of probability.

Look at the formula below as Professor K reminds you of the formula for calculating probability or ***chance***. After reminding yourself of the probability formula, look at **Example Set 1** for a review of how to use the probability formula:

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{elements in a sample set}}$$

Example Set 1

A box contains slips of paper numbered from *1* to *52*. *What is the probability of randomly drawing the number two from the box of numbers? What is the probability of drawing an even number from the box?*

To determine these probabilities Professor K must use the formula for determining probabilities and substitute the favorable ***outcome*** (drawing the number 2: one chance) into the numerator and the total number of possible outcomes into the denominator (52 numbers in the box).

$$\text{Probability} = {}^{1}/_{52}$$

Therefore, *one out of fifty-two* is the chance of drawing a number 2 from the box of numbers.

To calculate the next probability, Professor K must first realize that there are twenty-six even numbers in the box containing numbers 1-52. With this in mind, Professor K places the number of favorable outcomes, twenty-six (even numbers in the box) in the numerator and fifty-two, the number of possible outcomes (number of numbers in the box) in the denominator.

$$\text{Probability} = {}^{26}/_{52} = {}^{1}/_{2}$$

VOCABULARY

Chance: probability or possibility of something happening
Outcome: result of an experiment
Sample space: the set containing all possible outcomes

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Therefore, the probability of drawing a 2 from the box of numbers is *twenty-six out of fifty-two*. Professor K reduces the fraction as shown to obtain a *one in two chance* of drawing an even number.

In **Example Set 2** Professor K reviews probability with you using a number cube.

Example Set 2

A number cube consisting of twelve sides is labeled with one of the following letters on each side: A, A, B, B, B, B, B, C, C, D, D, D. *Determine the probability of each letter ending in the "up" position after rolling the cube*. Professor K first determines the value of the denominator of the probability formula to be twelve since there are twelve sides to the number cube and therefore, twelve possible outcomes. Next, Professor K counts the number of times each letter appears in the ***sample space*** and places that count in the numerator of the probability formula to calculate the probability of rolling that letter. Follow Professor K's calculations below as he calculates each of these probabilities.



Number of *A*s = 2 probability (rolling A) = $^{2}/_{12} = {}^{1}/_{6}$
Number of *B*s = 5 probability (rolling B) = $^{5}/_{12}$
Number of *C*s = 2 probability (rolling C) = $^{2}/_{12} = {}^{1}/_{6}$
Number of *D*s = 3 probability (rolling D) = $^{3}/_{12} = {}^{1}/_{4}$

Professor K's son, Matthew, had a cup in which he placed several colored marbles. Look at **Example Set 3** as Professor K reviews more questions of probability with you using Matthew's cup of marbles.

Example Set 3

Matthew pulled a green marble from the cup that contained two red, two green, two black, one white and two blue marbles. He placed the green marble back in the cup and proceeded to pull a white marble from the cup. Matthew did not return the white marble to the cup. Matthew then pulled a black marble from the cup. *Determine the probability of Matthew pulling a green marble, a white marble and a black marble from the cup*.

Step 1 Calculate the probability of each outcome separately. Remember there are 9 elements in the sample space.

$$P(\text{green}) = {}^{2}/_{9}$$
$$P(\text{white}) = {}^{1}/_{9}$$
$$P(\text{black}) = {}^{2}/_{8} = {}^{1}/_{4}$$

(Remember, the number of elements in the sample space was reduced by one, because Matthew did not return the white marble to the cup.)

Step 2 Calculate the probability, P(green, white, black).

$$P(\text{green, white, black}) = {}^{2}/_{9} \bullet {}^{1}/_{9} \bullet {}^{1}/_{4} = {}^{2}/_{324} = {}^{1}/_{162}$$

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Professor K multiplies the individual probability for each color marble together to obtain the probability that Matthew pulls a green, white, and black marble from the cup under the conditions given.

Show Professor K that you understand how to work probability problems by working the activities for this lesson.

LIFE PRINCIPLE



"The probability of success is determined by your attitude."

—Anonymous

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Intermediate Math Skills



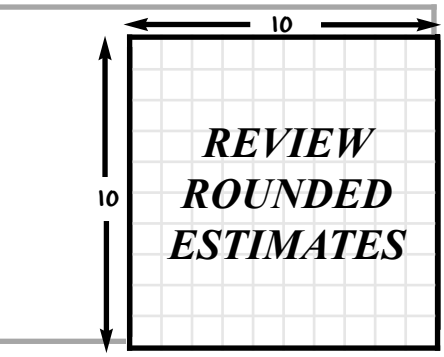
Lesson 13

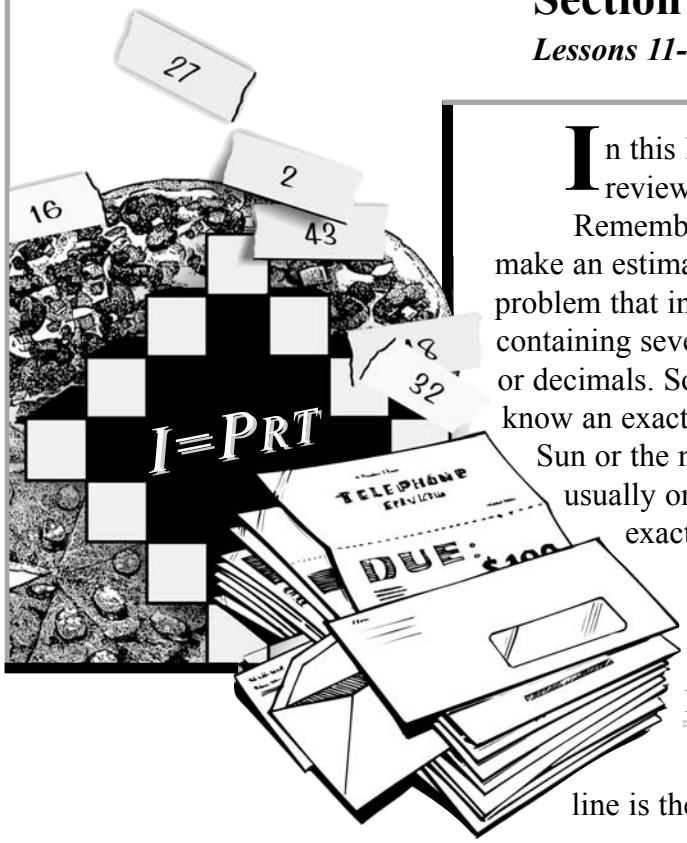
Predictions

Chapter 3

Section 3

Lessons 11-15





In this lesson Professor K will now review ***rounding*** and ***estimating***.

Remember that *rounding* allows you to make an estimation of the answer to a problem that involves whole numbers containing several digits, as well as fractions or decimals. Sometimes you do not have to know an exact answer to a problem. The distance to the Sun or the number of people living in the world are usually only approximations or estimates of the exact answer.

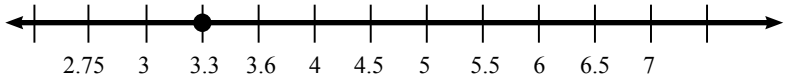


Professor K

Look at **Example Set 1** as Professor K shows you how to make an estimation of the square root of a number.

Example Set 1

Determine which number on the number line is the best estimate of $\sqrt{11}$.



Professor K knows that $\sqrt{9} = 3$ and that the $\sqrt{16} = 4$. Therefore, $\sqrt{11}$ should be located between *3* and *4* on the number line. Looking at the number line and the possible answers, Professor K determines that 3.3 represents the closest estimation for $\sqrt{11}$.

VOCABULARY

Rounding: a mathematical procedure used to make estimations of an outcome

Estimation: a calculated guess

Look at **Example Set 2** as Professor K works a problem involving *rounding*.

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Example Set 2

Professor K wanted to take his math class to the local pizza parlor to reward them for doing so well on the last math test. Professor K knew that the pizza parlor had an "all-you-can-eat" special that costs $4.65 for each student. Professor K wanted to estimate the amount of money he should take to pay for pizza for 21 students. Professor K wanted to make a quick calculation in his head so he rounded the price per student, $4.65 up to $5.00 (.65 is closer to the next larger whole number, 5, than to 4). Professor K could also round the number of students, 21 down to 20 (rounding to closest five, 21 is closer to 20 than 25). Professor K then performs the following calculation in his head.

$$5 \bullet 20 = 100$$

Professor K knew quickly that he needed at least $100.00 to purchase pizzas for his class.

In **Example Set 3**, Professor K estimates the amount of simple interest he will pay on a loan from his bank.

Example Set 3

Professor K borrowed $57,000.00 from his bank at an interest rate of 4.75%. Professor K wanted to estimate in his head the amount of simple interest he would pay the first year. Professor K remembered the formula for simple interest, $I = Prt$. Professor K then rounded $57,000.00 up to $60,000.00 and the interest rate to 5%. In his head Professor K then multiplied $60,000 ($P$) times .05 ($r$ = 5% = .05) times 1 (t = 1 year).

$$I = (60{,}000)(.05)(1) = 3{,}000.00$$

The estimated simple interest Professor K will pay the first year on a loan of $57,000.00 at a 5% rate of interest is $3,000.00.

Show Professor K that you understand how to use rounding and estimating to simplify calculations by working the activities for this lesson.

Life Principle



"Estimate the cost of your actions."

—Anonymous

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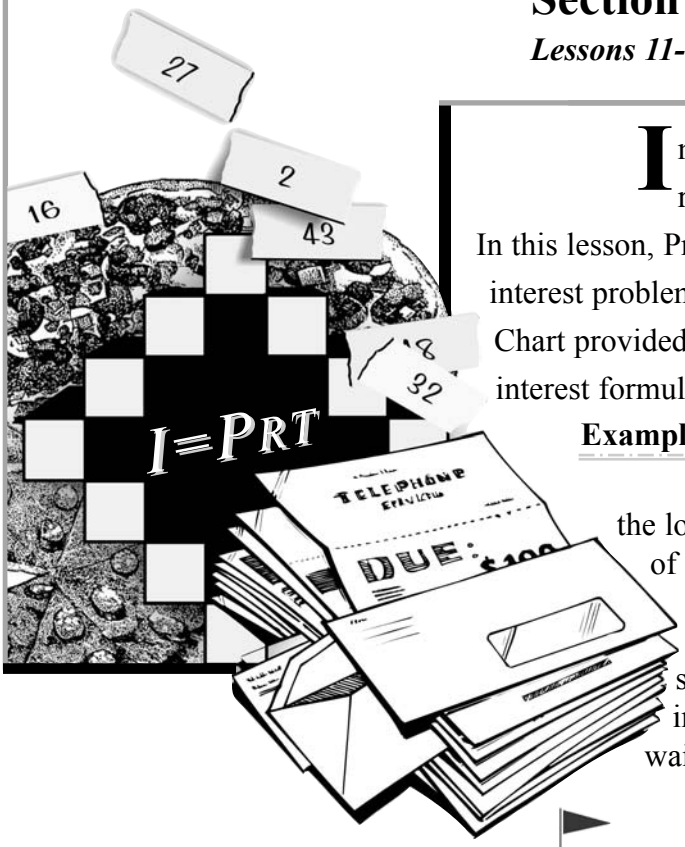
Lesson 14

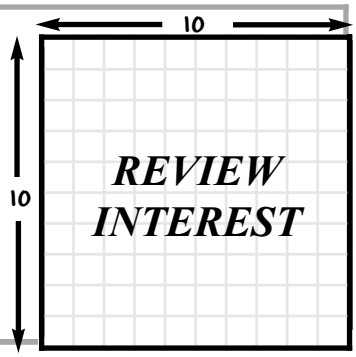
Percent

Chapter 3

Section 3

Lessons 11-15





In the previous lesson, Professor K reviewed some interest problems with you. In this lesson, Professor K will review more simple interest problems. Remember to refer to the Mathematics Chart provided at the end of this chapter for the simple interest formula and time conversions.

Example Set 1

Professor K borrowed $20,000.00 from the local bank for a ***term*** of 20 years. The rate of interest was 10% per year. Professor K wanted to calculate the amount of simple interest he would pay. Professor K will show you how to calculate the amount of interest that he would owe on his loan if he waited 20 years to ***pay off the loan***.

Step 1 $I = Prt$

In Step 1, Professor K writes the simple interest formula.

Step 2

$I = 20{,}000 \bullet .10 \bullet 20$

$I = 20{,}000 \bullet 2$

$I = 40{,}000$

In Step 2, Professor K places the principal, P of the loan, $20,000.00, the rate in decimal form, r = 10% = .10, and the time, t = 20 years into the simple interest formula. Professor K then performs the calculations to determine that $40,000.00 is the amount of interest that Professor K would owe on his loan at the end of 20 years.

VOCABULARY

Term: a period of time at which a loan is due and payable
Pay off the loan: statement used to mean repayment of a loan in full
Note: a loan



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Example Set 2

Mrs. K borrowed $10,000.00 from the local bank for a year. The rate of interest was 9% per year. Mrs. K was able to repay the ***note*** within eight months. Professor K will show you how to calculate the amount of interest that Mrs. K owed when she repaid the note.

Step 1	$I = prt$	In Step 1, Professor K writes the simple interest formula.
Step 2	$t = {}^{8}/_{12} = {}^{2}/_{3} = .67$	In Step 2, Professor K knows that the rate is calculated on a per year basis. Since Mrs. K repaid the loan in eight months, Professor K placed 8 months in the numerator of a faction with a denominator of 12 months (1 year = 12 months). Professor K divides out the units (months) and then calculates the decimal equivalent of ${}^{8}/_{12}$, which is .67.
Step 3	$I = 10{,}000 \bullet .09 \bullet .67$ $I = 10{,}000 \bullet .06$ $I = 600$	In Step 3, Professor K places the principal of the loan ($10,000.00), the rate in decimal form (9% = .09), and the time (.67) into the simple interest formula. Professor K then performs the calculations to determine that $600.00 is the amount of interest that Mrs. K owed on her loan at the end of 8 months.

Show Professor K that you understand how to work simple interest problems by working the activities for this lesson.

Life Principle



"Take note of the things that interest you, for they will affect the balance of your life."

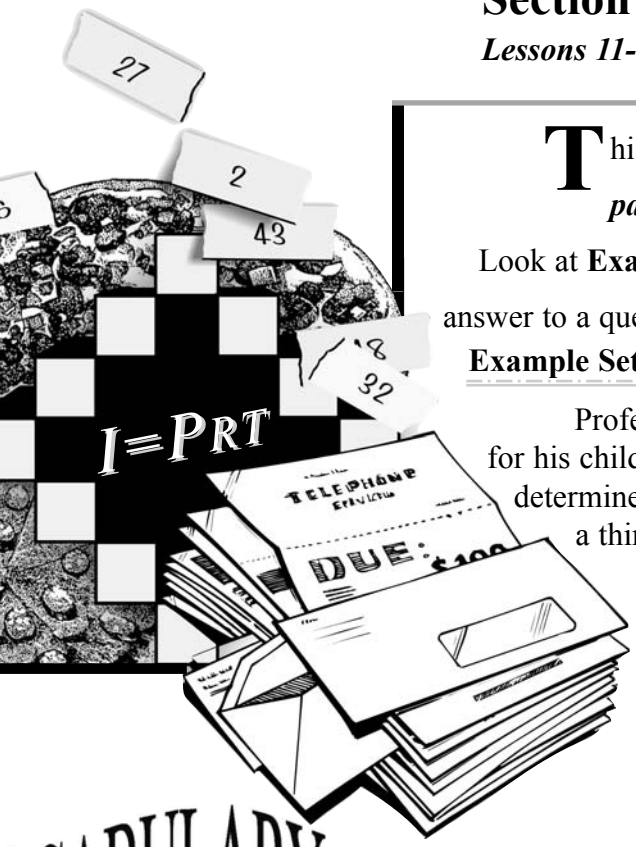
—Anonymous

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Chapter 3

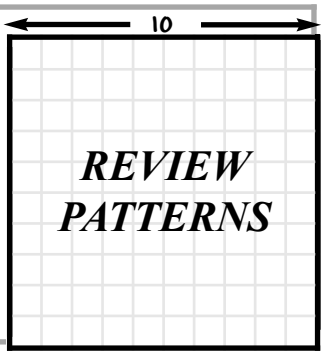
Lesson 15

Predictions



Section 3

Lessons 11-15



This lesson will be a review of ***patterns*** you studied in Chapter 7. Look at **Example Set 1** to determine the answer to a question posed by Professor K.



Professor K

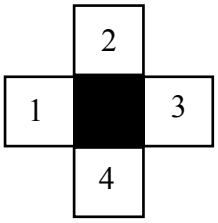
Example Set 1

Professor K drew a square on a sheet of paper for his children. Professor K asked his children to determine the number of squares necessary to form a third ***layer*** around the original square.

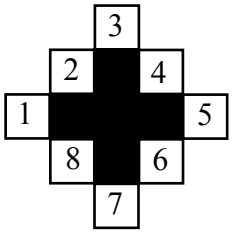
Original Square



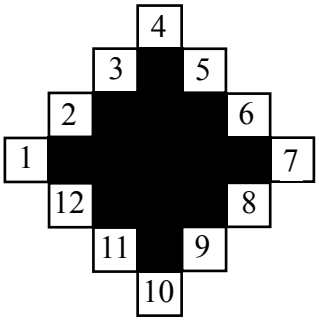
First Layer = 4



Second Layer = 8



Third Layer = 12



By adding squares to the ***exposed faces*** or sides of the previous layer (excluding the tops), Professor K's children determined that the number of squares in the third layer was 12 squares.

VOCABULARY

Patterns: arrangement of design, predictable sequence
Layer: the number of items necessary to cover all the exposed surfaces of a 3-dimensional (solid) object, or sides only of a 2-dimensional object (such as a square)
Exposed faces: all faces (sides, top, bottom) of a solid shape that are not covered

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Professor K asked his children to apply the information they learned in **Example Set 1** to answer another question in **Example Set 2**.

Example Set 2

Professor K asked his children if they could determine a formula that could be used to calculate the number of squares in the *n*th layer for the pattern in **Example Set 1**. Professor K's children made the following table to see the pattern in the numbers.



First layer	4 squares
Second layer	8 squares
Third layer	12 squares

Professor K's children noticed that the number of squares per layer were all multiples of 4. They then noticed that to calculate the number of squares in the first layer could be obtained by multiplying $n = 1$ times 4. The number of squares in the second layer can be obtained by multiplying $n = 2$ times 4 and the number of squares in the third layer by multiplying $n = 3$ times 4. Therefore, the following would be the formula for the number of squares in the *n*th layer: $4(n)$ squares.

In **Example Set 3** Professor K encouraged his children to ask him a pattern question.

Example Set 3

Professor K's children showed Professor K a sequence of numbers. They wanted Professor K to tell them what the seventh term or number in the seventh position would be. Below is the sequence of numbers Professor K's children showed Professor K.

4, 10, 28, 82, …

Professor K looked at the numbers and made the following attempts to create a formula for determining the value of the term in the *n*th position.

Attempt 1: $n = 1 \rightarrow n + 3 = 1 + 3 = 4$; works for the first term.
$n = 2 \rightarrow n + 3 = 2 + 3 = 5$; does *not* work for second term.

Attempt 2: $n = 1 \rightarrow n \bullet 4 = 1 \bullet 4 = 4$; works for the first term.
$n = 2 \rightarrow n \bullet 4 = 2 \bullet 4 = 8$; does *not* work for the second term.

Attempt 3: $n = 1 \rightarrow 4^n = 4^1 = 4$; works for the first term.
$n = 2 \rightarrow 4^n = 4^2 = 16$; does *not* work for the second term.

Attempt 4: $n = 1 \rightarrow 3^n + 1 = 3^1 + 1 = 3 + 1 = 4$; works for the first term.
$n = 2 \rightarrow 3^n + 1 = 3^2 + 1 = 3 \bullet 3 + 1 = 9 + 1 = 10$; works for the second term.
$n = 3 \rightarrow 3^n + 1 = 3^3 + 1 = 3 \bullet 3 \bullet 3 + 1 = 27 + 1 = 28$; works for the third term.
$n = 4 \rightarrow 3^n + 1 = 3^4 + 1 = 3 \bullet 3 \bullet 3 \bullet 3 + 1 = 82$ works for the fourth term.

Professor K feels confident that his formula, $3^n + 1$ will calculate the value of the *n*th term. Professor K calculates the seventh term using his formula:

$$3^n + 1 = 3^7 + 1 = 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 \bullet 3 = 2{,}187 + 1 = 2{,}188.$$

Professor K reminds you that the formula for *patterns* and *sequences* is determined by a series of trial-and-error steps.

Show Professor K that you understand how to recognize patterns and determine formulas for patterns by working the activities for this lesson.

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Life Principle



"Pattern your actions after that which is good."

—Anonymous

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Intermediate Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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Mathematics Charts

PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$\textit{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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